

Supporting Mathematical Literacy Development: A Case Study of the Syntax of Introductory Algebra

Danielle Leibowitz
University of Illinois at Chicago

Existing research on how to develop students' mathematical literacy skills is limited and offers few explicit recommendations regarding verbal and visual cues that can be used by mathematics educators to assist their students in making the connections necessary to develop their fluency in mathematical language, particularly in regard to mathematical syntax. This study examined the methods used by one introductory algebra teacher to support her ninth grade students' mathematical literacy skills, specifically examining how she supported their understanding of the mathematical syntax of the distributive property as applied in algebra. Video footage of one class of ninth grade students from an urban high school was analyzed for the teachers' use of discourse and gestures to support her students' understanding. Results indicated a consistent pattern in the teachers' use of dialog, gesture and references to the algebraic expressions.

Introduction

A variety of indicators, including labor statistics, national assessment data, and international performance metrics indicate that there is a need to improve the outcomes of mathematics education in the United States. The U.S. Bureau of Labor Statistics, for instance, projects that from 2012 to 2022, there will be a 26.1% increase in mathematics occupations due to growth need, and that other mathematics-related occupations will also experience above-average growth, most of these positions requiring a bachelor's degree, and some of the fastest-growing requiring a master's degree (Richards & Terkanian, 2013). However, in 2014, only 43% of high school graduates were ready for college level mathematics, showing 0% improvement since 2010 (ACT, 2014). Additionally, even students who did successfully enter a mathematics or mathematics based collegiate program often did not complete their degree. Data from the National Center for Education Statistics (NCES) indicate that 40% of bachelor's degree students and 69% of associate's degree students who declared STEM (science, technology, engineering, and mathematics) majors between 2003 and 2009 either switched to a non-STEM major or left college without completing their degree by 2009 (Chen, 2013). International comparisons also indicate poor mathematics performance, ranking the United States 51st out of 144 countries in the quality of mathematics and science education (Schwab, 2014). In an international assessment conducted by the Organization of Economic Cooperation and Development (OECD) in 2012, 8.8% of 15-year-old students in the United States scored at proficiency level 5 or above (top performing), on the mathematics literacy portion of the assessment, which was lower than the OECD average of 12.6%, and 25.8% scored below level 2 (baseline proficiency), which was higher than the

OECD average of 23.0% (Programme for International Student Assessment, 2014). Additionally, the U.S. average score was 481, which was lower than the OECD average of 494. The analysis also indicated that these scores were not measurably different from those in previous years, dating back to 2003.

While there are numerous factors that influence student performance in mathematics, one that has received relatively little attention concerns the literacy skills that are necessary for doing and understanding mathematics. Mathematical literacy is defined as the ability to read, write, speak, and listen to mathematics with understanding (Thompson & Rubenstein, 2014), and only recently have educational policy documents begun to emphasize its role in mathematics education. For instance, the edTPA (a recently mandated performance-based assessment for pre-service teachers) outlines mathematical language demands that students are expected to develop, including knowledge of content-specific vocabulary, understanding of mathematical syntax and discourse, and the ability to use various language functions, such as conjecturing, explaining, and proving (Stanford Center for Assessment, Learning and Equity, 2014). Additionally, the Common Core State Standards indicate a number of standards that refer to mathematical literacy (e.g., making use of mathematical symbols, justifying conclusions, and communicating reasoning to others) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Traditionally, it was assumed that students could apply general reading and writing skills and strategies to any content area, and thus, strategies specific to mathematical literacy development were not a focus; however, research has since indicated that disciplines differ extensively in their fundamental purposes

es, symbolic artifacts, traditions of communication, and use of language, suggesting that a generalist approach to developing mathematical literacy may be ineffective (Buehl, 2011; Shanahan & Shanahan, 2012). As a result of these findings, there has been increased emphasis on identifying disciplinary literacy practices and developing instruction that supports students in acquiring discipline-specific skills. Through disciplinary literacy practices, students uncover the meaning behind the terminology and symbols used in the discipline, and learn to view the subject matter from an insider's perspective (Shanahan & Shanahan, 2012).

Building off the notion that literacy skills should be formed through a discipline-specific approach, various strategies for strengthening mathematical literacy have been developed. In mathematics, where words, symbols, and diagrams hold implicit and explicit meanings that students must connect and translate, there is a need for constant literacy instruction that is embedded into classroom routines (Gomez, Lozano, Rodela, & Mancevice, 2013; Shanahan & Shanahan, 2012; Thompson & Rubenstein, 2014). This can be done through discourse moves, such as waiting for students to respond after asking a question, revoicing a student's response to provide clarification or expansion, inviting students to participate by sharing varied solutions, probing students' thinking, and creating opportunities for students to engage with another's reasoning (Thompson & Rubenstein, 2014). Additionally, students must be taught to read through a mathematical lens (Buehl, 2011), and to attend to the precision of meaning that each word and symbol represents. This ability is likely to be enhanced by developing students' conceptual understanding alongside their language skills, since research points to a lack of conceptual knowledge as a major contributor to why so many students feel they cannot learn through mathematical texts. Additional strategies, such as direct mathematical vocabulary instruction, allowing students to create informational posters and memory guides (Edwards, Maloy, & Anderson, 2009), and various reading strategies, including the preview, predict, read, and review strategy, and the concept card strategy, have also been shown to be helpful in students' mathematical literacy development (Campbell, Schlumberger, & Pate, 2001).

While there exists a growing body of work on mathematical literacy development techniques, the topic remains drastically understudied, especially at the level of day-to-day instructional processes used by teachers. The present study is an exploratory case study of the instructional strategies used by one introductory algebra teacher to support her ninth grade students' understanding of key literacies involved in mathematical ideas central to algebraic reasoning. Specifically, this study focused on the mathematical syntax of the distributive property as applied in algebra. Algebra holds a unique position in mathematical literacy, because it is often where symbolic language is confronted seriously for the first time (Buehl, 2011), and the distributive

property is of particular interest because it is essential for algebraic functioning and has been shown to present difficulty for students (Boulton-Lewis, Cooper, Pillay, & Wilss, 1998; Demana & Leitzel, 1988). For example, given the expression $6(a - b) = 20$, students must be able to "read" the expression appropriately and create semantically equivalent expressions or transformations of the given expression. Acceptable "meaning preserving" syntactic forms (e.g., $6a - 6b = 20$ or $a - b = 20/6$) are often the first step to a solution. The central research question of this study focused on identifying specific instructional practices used by a ninth grade teacher to develop students' mathematical literacy skills in the area of the syntax of the distributive property.

Methods

Participants

One teacher from an urban high school was chosen to participate in this study. The teacher was Caucasian and had at least five years of teaching experience. The course videotaped was a double-period Algebra I class. Students were in the ninth grade, mostly African American, and demonstrated math achievement that indicated that they were underprepared for Algebra I.

Data Corpus Analyzed

Video footage of classroom instruction (9 hours and 20 minutes) was collected over two and a half months. Whole lessons were videotaped, thus providing footage of whole class discussion, teacher-student dialogue, student-student dialogue, and student work. The teacher was using the Intensified Algebra curriculum for the first time, which is aligned with the Common Core State Standards and promotes deeper learning (Agile Mind, 2009).

Initial Analytic Strategy

Each video was viewed to provide a general overview of the prevalence of instructional practices that might be supporting mathematical literacy elements. Based on this initial review, the search was narrowed to segments of lessons where the emphasis was on the syntax of algebra. Segments focusing on syntax were noted. Of particular interest for this case study are two lessons that occurred nine days apart, where there was a concerted focus on the syntax of the distributive property with algebraic expressions. The distributive property was of interest because it is a core syntax of algebra, and being able to create mathematically equivalent expressions is a key step in algebraic problem solving.

For these selected lessons, an intensive analysis was conducted of the teacher discourse and gestures coordinated with notations made on or connected to the terms in the algebraic expressions of the distributive property. The discourse was analyzed for words or phrases that occurred repeatedly when the distributive property was the focus of student work. The

discourse was coordinated with images of the algebraic expressions on the white board, which were examined for any patterns as well, including the structure of the solutions across the different examples, any frequently used notation or symbols, and words that may have been written alongside the notation.

Results

A consistent pattern of dialogue, gesture and references to the algebraic expression was noted across seven problems using the distributive property during the focal lessons of this case study. This pattern was observed in one segment that occurred during the first recorded lesson and was picked up and reiterated in six additional problems/segments during the second recorded lesson, which occurred nine days later. Two instructional trends were observed: use of an intermediate multiplication step (i.e., multiplying the number on the outside of the parentheses with the terms on the inside of the parentheses), and the use of informal symbols to support students' understanding of the process that occurs when distributing, specifically arrows that were used to illustrate the distribution. The arrow(s) were drawn while the teacher was verbally describing the distribution, or stating the multiplication that would occur as a result of the distribution. These trends are specifically illustrated in four segments.

Detailed Analysis

Segment 1. This segment took place during the first lesson and occurred in a group discussion on a problem from the students' homework the previous night. This homework assignment focused on writing algebraic expressions to represent the perimeter of a rectangle, and this specific problem included two potential processes for determining an expression, which students had to judge for accuracy. The dialogue here centers on discussing the error in John's work, as well as how to correct his mistake. The work on the board reflecting the representation created by the teacher as she walked through the problem is shown in Figure 1.

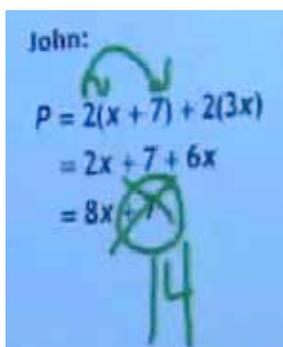


Figure 1. Problem focus for Segment 1.

As is evident in Figure 1, arrows were used to show distributing the “2” on the outside of the parentheses to both terms inside the parentheses. The transcribed discourse illustrates how this representation developed through the coordinated use of verbal and nonverbal communication mechanisms.

Teacher: How did he get this seven wrong? [circles the “7” in the last line of John’s work]

Student 1: He forgot to add the other seven.

Teacher: Close, you’re right he forgot to add the other seven. Remember, this is that mistake that happens with the distributive property. So, I got this two outside [points to the “2” in the first line], so I’m doin’ the two times x [draws an arrow from the “2” to the “x”], but what did he forget to do?

Class: Two times seven.

Teacher: Two times that seven [draws an arrow from the “2” to the “7”]. This is that mistake we’re talking about with the distributive property, that’s what he made. He forgot to distribute the two [points her pen toward the “2” in the first line] to the seven [points her pen toward the “7” in the first line]. Two times seven will give you a fourteen [writes “14” on the board below the “7” in the last line], which is what it should be [crosses out the “7” in the last line].

Rather than simply stating the error (failing to recognize that the “2” was the multiplier for each of the terms in the parentheses, emphasizing the “7”), the teacher’s use of arrows creates a visible trail showing this distribution explicitly and in a way that remained visibly present for the remainder of the lesson.

Segment 2. The problem shown in this segment took place in the second lesson and was one of six problems as part of an activity on determining equivalent expressions. Here, the teacher is demonstrating how to simplify the expression on the right side of the equation, which requires the use of the distributive property. This problem served as an example problem before students began work independently on the other five problems, many of which also required the distributive property. The work of the teacher as she guided students through this problem is shown in Figure 2.

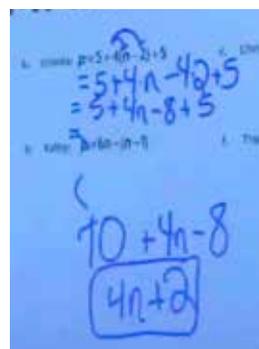


Figure 2. Problem focus for Segment 2.

As illustrated in Figure 2, similar to Segment 1, arrows were used to show the distribution of the “4” on the outside of the parentheses to both terms inside the parentheses. Additionally, an intermediate step, showing the multiplication that follows

from the distribution, was shown in the second line. The transcribed discourse again illustrates the coordinated use of verbal and nonverbal communication mechanisms.

Teacher: Now what are the directions they want you to do?

Student 1: Rewrite the form.

Teacher: Rewrite in what kind of form?

Student 1: Simpler form.

Teacher: Simpler form. What that means is they want you to collect like terms, or combine like terms. Okay? So we have to do distributive property, when we have to do it, and we have to combine terms that are alike.

...

Teacher: How, if I want to combine things, order of operations, I still have to get rid of these parenthesis first, okay [waves her pen around the " $4(n - 2)$ "]? 'Cause there's this four on the outside [points to the "4" in the first line with her pen]—

Student 1: It's just gon' be "n".

Teacher: That means I have to distribute this four to everything that's on the inside of those parenthesis [draws an arrow from the "4" to the "n", and an arrow from the "4" to the "2"]—

Student 1: Four-n minus eight.

Teacher: So, I'm going to bring down my five, four times n [writes " $= 5 + 4*n$ " on the board below the first line], now this minus sign—

Student 1: That's negative eight.

Teacher: I just bring it down, or you can add a negative eight. Okay, so four times two, plus five [writes " $- 4*2 + 5$ "]. So then we have to do one more step—

Student 1: You gotta multiply the four and the two.

Teacher: So I have five plus four-n, minus eight, plus five [writes " $= 5 + 4n - 8 + 5$ "]. And what is five plus five?

Class: Ten.

Teacher: Ten. I'm gonna run out of room, I'm gonna jump down here so you can see me [draws an arrow from previous work to where she is about to write]. So I have ten, plus four-n, minus eight [writes " $10 + 4n - 8$ "]—

Student 1: Ten minus eight.

Teacher: Okay, and I can keep going 'cause I can combine these. What's ten minus eight?

Class: Two [writes " $4n + 2$ "].

Teacher: So this would be putting it in simpler form [puts a box around the final answer]. Okay? So, now I showed all

of the steps, okay, I do want you to show your work, but I showed every single step so you could understand what we are doing, so if you are able to combine a couple steps at once, you can do that, but I want to see your work here.

In this segment, the teacher explicitly walked through the intermediate steps of transforming the algebraic expression to its simplest form. This process makes visible to students how these expressions change from their starting forms to their simplified forms. In particular, the teacher showed the multiplication that resulted from the distribution, and the teacher also continued the use of arrows to illustrate the distribution.

Segment 3. This segment occurred in the second lesson, after the students worked independently on the remainder of the exercise introduced in Segment 2. Students had the opportunity to work through this problem as a part of that exercise before going over it as a class. While going over this problem, the teacher reiterated the process demonstrated in Segment 2, again making explicit the distributive property operation through the use of an arrow.

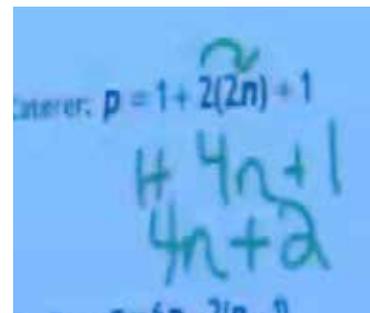


Figure 3. Problem focus for Segment 3.

As shown in Figure 3, arrows were once again used to express the distribution of the number outside of the parentheses, in this case "2", with the term inside the parentheses, "2n". The intermediate step of multiplying the "2" and "2n", however, was not explicitly shown. The transcribed discourse below shows the coordination of discourse and gestures from this segment.

Teacher: What do I have to do with number one to start to simplify that? What would be my first step?

Student 1: One plus two times two-n.

Teacher: Okay, so you're saying, out of all of that, what do I have to do? How do I get rid of those parenthesis?

Student 1: Put two times two-n.

Teacher: Okay [draws an arrow from the "2" to the "2n"], so, what is two times two-n?

Student 1: Four-n.

Teacher: Four-n. Okay, so I'm bringing down my one plus, plus one [writes " $1 + 4n + 1$ "]. So from there—

Student 2: Two plus four!

Teacher: One plus one is two, I cannot combine that because this is a variable, four-n [writes " $4n + 2$ "], and that is the answer.

This segment shows the teacher taking the students through the process of simplifying this expression, which she begins by asking the students how they would start, and continues by asking students how they would progress through the process, making corrections as necessary. The teacher also continues to use arrows as a visual representation of the distribution process, however, she does not explicitly show the intermediate step of multiplying the "2" and "2n".

Segment 4. This segment took place in the second lesson, approximately 20 minutes after the problem featured in Segment 3. This problem was taken from a latter part of the activity featured in Segments 2 and 3, and focused on the same skills of identifying equivalent expressions and using the distributive property. Students had time to work on this problem independently before discussing it as a class. One student specifically requested to look over this problem as a group, which the teacher responded to by going over how to think of the problem conceptually, followed by how to simplify the expression using distribution. The original expression was: " $12 - (x + 9)$ ", and thus, when giving the conceptual explanation, the teacher described how one could think of subtracting out everything that is inside of the parenthesis, meaning that the "x" and the "9" would be subtracted from the "12" leaving " $12 - x - 9$ " (which the teacher wrote on the board). Immediately following, the teacher presented the work seen in Figure 4, and discussed how to approach this problem by distribution, which is seen in the coordinated discourse and gestures that follow.

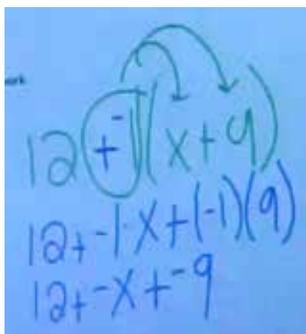


Figure 4. Problem focus for Segment 4.

Figure 4 shows the teacher's use of arrows to illustrate the process of distribution, which we have seen in all of the preceding segments. This figure also shows the teacher's use of an intermediate step, making the multiplication of the number outside of the parentheses with the terms inside of the parentheses explicit.

Teacher: Another way to think about this is, what number is in front of that parenthesis even though it's not written?

Student 1: One.

Teacher: It's one, have you guys heard of that before?

Student 2: Nope.

Teacher: No, okay, um, so there is...there's like this minus one [writes "-1" next to the "12"], it might not be written, but it's like an imaginary one. And the reason why [writes " $(x + 9)$ " next to the "-1"], because if I just have a variable [writes an "x" off to the side], even though there's no number in front of it, there is a number that's right there [writes a "1" in front of the "x"]. And the reason why they don't do it is, if I multiply anything by one, you're still going to be left with what you multiplied, right? Five times one is going to be five. So here, I'm really distributing this [circles the "-1" in the first line] to everything in the parenthesis [draws an arrow from the "-1" to the "x", and from the "-1" to the "9"]. So I would change this minus to an addition problem, and if I change it to addition [changes the negative sign in front of the "1" to a plus sign], this positive one changes to a...?

Student 3: Negative one.

Teacher: Negative one [writes a small negative sign toward the top of the "1" in the first line]. So I'm distributing a negative one to x [writes " $12 + -1*x +$ "], and I'm distributing this negative one to this nine [writes " $(-1)(9)$ "]. So a negative one times x is a negative x, and a negative one times nine is a negative nine [writes " $12 + -x + -9$ "].

This segment reiterates the patterns that we have seen in the previous segments, specifically the use of arrows to illustrate the process of distribution and the use of an intermediate step, seen in the second line, to explicitly demonstrate the multiplication that results from the distribution. This problem is unique from the others in that it is the first problem that involved distributing a negative; however, the teacher's process of simplifying the expression remained consistent with the methods used in the other segments, especially with regard to the use of arrows and intermediate steps.

Discussion

This case study illustrates the use of verbal and nonverbal communication techniques to support students in understanding the syntax of the distributive property. Two recurring practices were noted, specifically, the use of informal notation (arrows) and intermediate steps (illustrating the multiplication following distribution). The instructional strategies involved explicit modeling of the transformations and explicit verbalization of the reasons that supported the specific transformations and why they preserved the equivalence of the original expression. This case study also shows the teacher listening to the responses of students as important cues to what features of the syntax

and its transformation needed to be explicitly explained. The case study suggests several instructional strategies that may be helpful in the mathematical literacy involved in transforming expressions into other equivalent forms.

Although only one exemplar, the systematic nature in which these practices were applied suggests that they would have an impact on student understanding. A limitation of the present study is that data on student performance were unavailable, and therefore this remains an open question that additional research needs to address. Furthermore, future research is needed on more examples of specific instructional methods for enhancing students' mathematical literacy skills in algebra as well as in other mathematics. Considering that some methods used by teachers might be minimally effective or even pose confusion, it is valuable to know which verbal and visual cues impede student learning and which advance learning. Assuming that the results of future research suggest promise in the systematic use of practices that make explicit how to correctly interpret and use mathematical syntax, the work has implications for teacher preparation programs, continuing education programs, and professional development.

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Danielle Leibowitz graduated from the University of Illinois at Chicago in May 2015 with a Bachelor of Science in the Teaching of Mathematics. Danielle was a member of the College of Liberal Arts & Sciences and the Honors College, and was heavily involved in student leadership, primarily through her role as the Student Member of the Board of Trustees during her junior and senior year. By developing her understanding of leadership, research, and the many structures that impact education, Danielle hopes to someday influence educational policies. Currently, Danielle is teaching high school mathematics in Chicago.

